

The M_4 Shallow Water Tidal Constituent From Altimetry and Tide Gauges.

Ole B. Andersen

Kort- og Matrikelstyrelsen, Geodetic Division, Rentemestervej 8, Copenhagen NV, Denmark

oa@kms.dk

Abstract. Present global ocean tide models based on the TOPEX/POSEIDON satellite data are still not accurate enough on the continental shelves for detailed oceanographic studies. The need for increasing accuracy in shelf regions calls for inclusion of more than just semidiurnal and diurnal constituents in future global ocean tide models.

This study is an extension to previous investigations on the Northwest European Shelf by (Andersen, 1999) which demonstrated that the major non-linear shallow water tidal constituent could be mapped from T/P altimetry. The purpose of this work is to locate in which regions of the world the M_4 shallow water constituent is of importance to tidal modelling and to advise where it should be included in future tidal models.

Maximum amplitude of more than 10 centimetres are found at the Newlyn tide-gauge in the English Channel. Significant amplitudes are not just concentrated to the Northwest European shelf but is also found at several other shelves like the Patagonian Shelf and several coastal shelves around the Atlantic Ocean..

Keywords. Altimetry, shallow water tides, tide gauges.

1 Introduction

Accurate sea level observations from the TOPEX/POSEIDON (T/P) satellite has vastly improved the global ocean tide models and numerous accurate global tide models are now available (i.e., PODAAC, 1996). In the deep ocean, present global ocean tide models agree within 2-3 cm in total (Shum et al., 1997; Smith and Andersen, 1997).

It is considerably more difficult to model tides in shallow water. The spatial pattern of the tidal waves is scaled down over the shelves, first of all, because the tidal wavelength is proportional to the square root of the depth. Resonant or near-resonant responses add to the complexity of the tidal pattern and produce some of the worlds largest tidal amplitudes, such as

in the Irish Sea, the bay of Mount St. Michel, and in the Gulf of Maine / Bay of Fundy. Consequently major differences are still found between most global ocean tide models, and different models seem to perform better in certain regions.

Shallow water constituents are largely dependent on the bathymetry, the shape of the shelf, and the regional tidal regime. Wide shelves like the northwest European shelf will have totally different shallow water characteristics than narrow shelves (e.g., the east coast of the United States near Cape Hatteras). Similarly, shelves dominated by semidiurnal tides will have shallow water tides related to semidiurnal constituents whereas shelves dominated by diurnal tides will have another suite of shallow water tides related to the interaction between the diurnal constituents.

For most tidally influenced and heavily populated coastal regions numerous accurate local hydrodynamic models already exist. One example is the NW European shelf where the tides have been modelled for decades. For this shelf regional models that includes shallow water constituents are available from Flather (1976, 1981), Le Provost (1991), Chabert d'Hires and Le Provost (1979), Pugh and Vassie (1980), Walters and Werner (1991), Howrath and Pugh (1983), Pingree and Maddock (1978), Pingree and Griffiths (1979), and Davies (1986).

For many shelves of the world no investigations have so far been made of the shallow water constituents due to the lack of tide gauge observations. Satellite altimetry can be used to advise where local tide models should include shallow water constituents.

M_4 was chosen for this investigation as it is the largest shallow water constituent in most places of the world. This is naturally so, as it related to the M_2 constituent which is the dominant tidal constituent in most places of the world.

The paper is structured such that section 2 gives a description of the characteristics of shallow water tides. Section 3 presents an investigation of 118 World Ocean Circulation Experiment (WOCE) on-line tide gauges. Section 4 deals with altimetry for mapping shallow water constituent and possible problems with alias periods and mesoscale

variability. Finally, section 5 presents the M_4 shallow water constituents from altimetry and compares with the results from the WOCE tide gauge observations.

2 Shallow Water Tides

The total tidal variation h from any number of constituents can conveniently be expressed using a summation of the individual constituents amplitude A_i and phase g_i or sine coefficients U_i and cosine coefficients V_i such as

$$h = \sum_{i=1}^{i=n} A_i \cos(\sigma_i t - g_i) = \sum_{i=1}^{i=n} U_i \cos(\sigma_i t) + V_i \sin(\sigma_i t), \quad (1)$$

where g_i is the phase lag of the tide at Greenwich and σ_i is the angular speed of the i th constituent (Knudsen, 1993; Pugh, 1987). The summation is carried out over the n largest tidal constituents independent of the origin of these (astronomic or non-linear shallow water).

In the deep ocean the dynamics are more or less linear and the tide is adequately described using a number of diurnal and semidiurnal constituents. In shallow water the dynamics become nonlinear and the tidal spectra appears more complicated, and consequently a number of additional constituents must be included to model the variations in sea level. These constituents appear of a number of frequencies like diurnal, semi-, third- and quarter diurnal. At some location even higher diurnal like sixth, eighth and twelfth diurnal are seen.

Nonlinear distortions cause compound and overtides often referred to as shallow water constituent. The nonlinearities appear through the quadratic term of bottom friction, spatial advection and mass conservation. All of these interactions can conveniently be expressed as simple harmonic constituents with angular speed being multiples, sums, or differences of the frequencies of the well-known astronomical constituents (e.g., M_2 and S_2). Detailed description on shallow water tides can be found in i.e. Pugh, (1987); Le Provost, (1991) and Andersen, (1999).

The interaction of M_2 with itself will generate M_4 and higher harmonics like M_6 , M_8 and M_{12} . Shallow water constituents have generally small amplitudes, even in shelf regions. However, the M_4 shallow water constituent can be resolved from T/P altimetry as demonstrated by Andersen (1999).

3 Tide Gauge Estimates of M_4

One year of sea level observations from 118 World Ocean Circulation Experiment (WOCE) on-line tide gauges was analysed using the Proudman Oceanographic Laboratory Tidal Prediction Software (POLTIPS).

The amplitudes of the M_4 shallow water constituents at the 118 WOCE tide gauges are shown in Figure 1. For each tide gauge the amplitude is shown using a filled circle. The size of the circle is dependent on the amplitude at the gauge and 5 mm correspond to 10 cm. By using observations from several independent years of the WOCE tide gauge data it was possible to give an estimate of the consistency of the estimated constituent. Determining independent amplitudes and phases for 10 years accessed the accuracy of the M_4 constituent from tide gauges. This yielded a standard deviation of roughly 7% of the amplitude for amplitudes greater than 5 cm. For the phase the standard deviation was around 15-20 degrees.

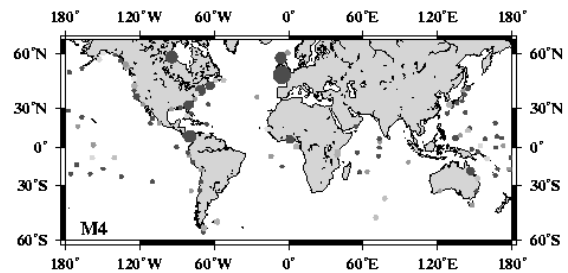


Fig. 1 Location of 118 WOCE tide gauge. The size of the circle is dependent on the amplitude at the gauge. A diameter of 3 mm correspond to 10 centimeters.

The M_4 constituent has mean amplitude of 1.5 cm for the 118 gauges and maximum amplitude in excess of 10 centimetres (12 cm) in the English Channel at the Newlyn gauge. Amplitudes larger than 6 cm are also found at several locations along the east coast of the US, on the Patagonian Shelf and in the Gulf of Benin.

An investigation of 118 tide gauges with far from a homogenous distribution along the world's shorelines is not representative for the shallow water tides of the world. The databases of the International Hydrographic Organisation contains much more than 1000 tide gauge stations and this would probably yield a much more detailed result. However, the 118 WOCE tide gauges are readily available on the Internet in a convenient and consistent format for the current analysis.

4 Altimetry and Shallow Water Tides

The ocean altimeter T/P Pathfinder product (version 8 containing 300 repeats) was used for this study. The NASA pathfinder altimetric observations were delivered with a set of provided standard corrections (see <http://iliad.gsfc.nasa.gov/ocean.html> for details).

The NASA Pathfinder products are delivered as normal point data, containing stacked, along-track observations. A special version without tidal correction was provided for this study (B. Beckley, personal communication).

The major astronomical tides were removed using the FES94.1 hydrodynamic model (Le Provost et al., 1994), which contains 13 constituents (K₁, O₁, P₁, Q₁, M₂, S₂, K₂, N₂, l₂, t₂, nu₂, μ₂, 2n₂). The FES94.1 model was chosen because it is pure hydrodynamic model without altimetry. Even though the FES94.1 model performs less accurate than newer models (i.e., GOT99.2 (Ray, 1999)) that rely on T/P altimetry, it was chosen because these models have already been fit once to T/P.

When using T/P sea level observations for mapping shallow water constituent special attention must be made to the ground track distance, which will limit the spatial resolution. The alias periods are also important as they might cause constituents to be inseparable from each other.

4.1 Ground Track Distance

The ground track distance or the distance between parallel tracks will result in a somewhat irregular distribution of the sea level observations. The inclination of the satellite is roughly 66°, which means that the satellite will cover only shelves within the 66° parallels. The ground tracks distance is 285 km at the Equator decreasing toward higher latitudes. At 50° latitudes, the ground track distance has decreased to roughly 150 km. Consequently, the spatial structure of shallow water tides can be more accurately mapped at high latitude.

4.2 Tidal Aliasing

Owing to the 10-day sampling interval of the T/P satellite being much longer than the tidal period, the tidal constituents are observed as having much longer periods (Parke et al., 1987, Andersen, 1994, 1995). The alias periods of the major four diurnal and

semidiurnal constituents are 62, 59, 50, 87, 173, 46, 89, 69 days for M₂, S₂, N₂, K₂, K₁, O₁, P₁ and Q₁, respectively (Parke et al. 1987, Andersen and Knudsen, 1997).

The alias period of the M₄ constituent is 31.04 days and the angular speed is 58 degrees/ hour corresponding to 6.1 hours between adjacent “high waters”. The M₄ constituent is well separated from all other important shallow water and astronomical constituents from T/P altimetry.

4.3 Applied Method

The shallow water constituent can be resolved using harmonic methods (i.e., Munk and Cartwright, 1966 or Knudsen, 1993). The annual variation in sea level has average amplitude between five and ten centimetres in most of the world (Knudsen et al., 1996). It was included to avoid any spectral leakage into the tidal estimates, as it has significant amplitude compared with the amplitude of the shallow water constituents.

Subsequently the tidal parameters were interpolated onto a regular grid of 0.5° latitude by longitude using optimum interpolation or collocation using a second order Gauss-Markov covariance function following Andersen, (1999).

The satellite estimates the elastic ocean tide and not bottom relative ocean tide as measured by the tide gauges. The elastic ocean tide is the sum of the ocean and load tide (e.g., Andersen 1995). To a first approximation, the load tide can be approximated by roughly -7 % of the altimetric tidal amplitude. Consequently, the satellite-derived estimates were corrected for this effect by multiplying the satellite-derived amplitudes by a factor of 1.075 to obtain the bottom relative shallow water ocean tide.

5 Results

The global distribution of amplitudes and phases of the M₄ constituent is shown in Figure 2. The results from T/P altimetry shows that the M₄ constituent has large amplitudes in places like the Northwest European shelf region. The Bay of Benin (western coast of Africa) also has large M₄ constituent, which is also seen from the WOCE gauges. Altimetry also reveals large amplitudes on most of the Patagonian shelf and in many local regions along the Southeast Asian coast and in the Yellow Sea M₄ can also be found.

M₄ from T/P altimetry

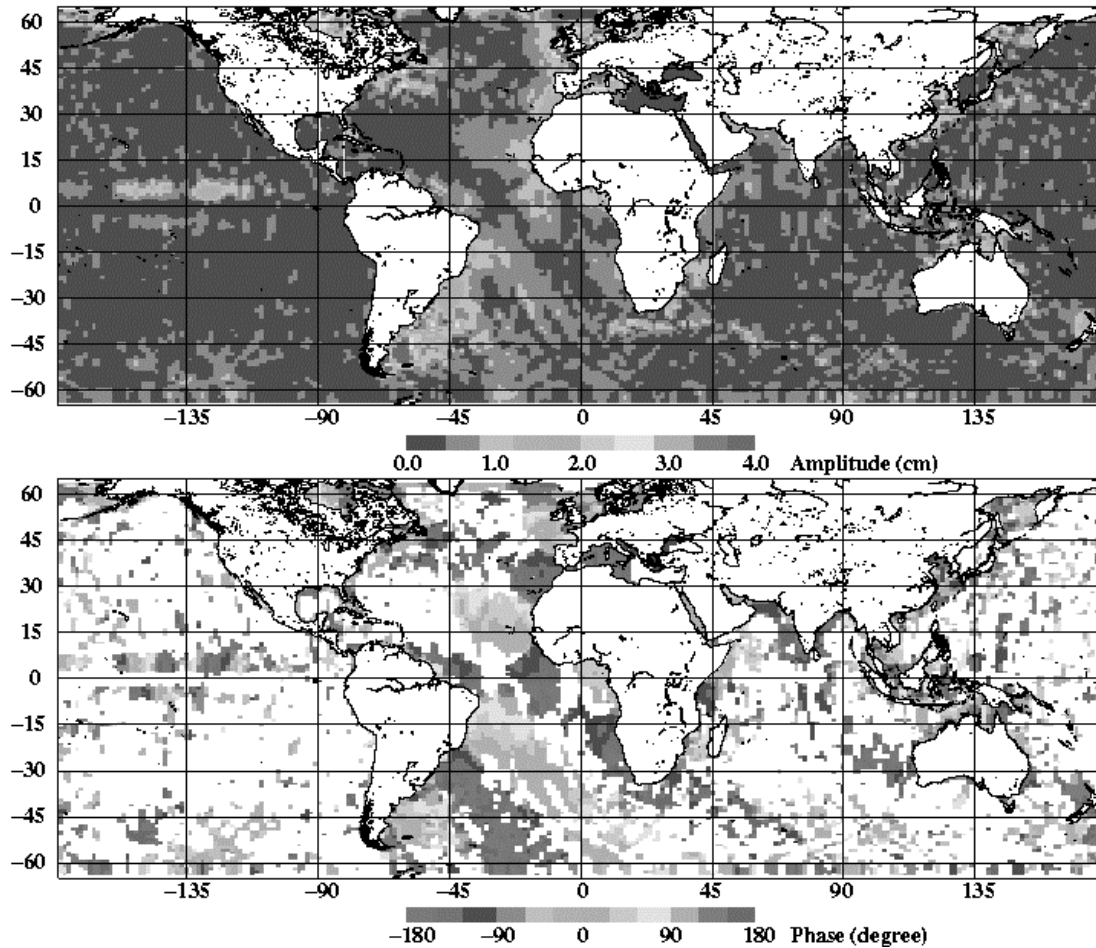


Fig. 2 The M_4 constituent from T/P altimetry data. The amplitudes are shown in the upper panel, and the phases in the lower panel. Amplitudes are in centimeters, and phases are in degrees with respect to Greenwich. In deep ocean areas (greater than 1500 meters) the amplitudes have been masked where the amplitude is smaller than 0.5 cm

It is interesting and quite unexpected that small amplitudes are also found in large parts of the deep Atlantic Ocean. The existence of such small amplitudes in the deep ocean was also found recently using a hydrodynamic model by Lyard and colleagues (Lyard Personal communication).

Regions of smaller M_4 amplitudes are found to be associated with the outlet of the Amazon River, in the Hudson Bay and in the Bay of Carpentaria between Australia and New Guinea. Amplitudes of around one centimetre were found at several locations in the South Atlantic Ocean. These are not fully understood, but it is assumed that they are related to mesoscale oceanic signal.

For the M_4 constituent large amplitudes are found by some WOCE tide gauges, but not by the altimetry.

There are several explanations for this. The most likely is the fact the high amplitudes can be related to a very local phenomenon. Consequently it is simply too localized for the altimetry to capture it. This is the most likely explanation to the findings at gauge in Panama, and one of the gauges along the Canadian west coast, which have considerably amplitude for M_4 whereas this is not captured by the satellite.

6 Summary

The main goal of this investigation was to determine if coherent characteristics of the major shallow water tidal constituent M_4 could be derived empirically from T/P altimetry. The purpose was to locate regions of the world, where this shallow water

constituent was found.

The empirical model gives an idea about the extent and amplitudes of the M₄ shallow water constituent. Due to the distance between the altimetric observations these models will only have limited skill in mapping shallow water constituents, except perhaps at very high latitude (Andersen, 1999). Subsequently these maps could be used to determine where local tide models should be established. One could subsequently derive an accurate shallow water tide model, by assimilating the altimetric derived constituents (Egbert, 1997) into regional/global hydrodynamic models in the future to provide accurate tide models, which must include shallow water constituents.

Coherent results between 118 WOCE tide gauges and M₄ was found in most regions. Several interesting features were resolved for this constituent by the satellite altimeter. Among these are significant amplitudes in many coastal parts surrounding the Atlantic Ocean, in South East Asia and on the Patagonian shelf. Further investigations should be carried out to investigate the influence of possible aliasing of mesoscale oceanic variability and to investigate the propagation of shallow water tides into the deep ocean.

With longer time series and the launch of the T/P follow-on satellite, JASON further development within shallow water tide modelling from altimetry can be expected. Results from the interlaced ground tracks data are going to be extremely interesting for those studying tides in general, but shallow water tides in particular.

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